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The application of stress path and critical state analysis to sediment deformation

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Abstract—The importance of consolidation, compression and shear as deformation mechanisms in uncemented sediments is discussed and the critical state concept is introduced as a unifying model for these aspects of deformation. From the critical state and the law of effective stress, the concepts of burial and tectonic stress paths are introduced with reference to the development of growth faults in an unlithified sediment undergoing simultaneous burial and extension. It is demonstrated that particulate deformation mechanisms can be important at burial depths of several kilometres, especially if the sediment is overpressured, and that deformation of this type will influence the geometry and nature of the structures produced. A model for the geometry and spacing of growth faults developed in unlithified sediments is proposed utilizing stress path and critical state concepts.

INTRODUCTION

THE EXCELLENT discussion of 'soft-sediment deformation' by Maltman (1984) has highlighted the problems of defining this term. Furthermore, certain mechanisms discussed by Maltman (1984) as being important in producing structures in 'soft-sediments' under shallow burial, can be shown to be important deformation mechanisms during deep burial and the tectonism of sediments. These deformation mechanisms are described in the soil mechanics literature as consolidation, compression and shear (Henkel 1960, Atkinson & Bransby 1978, Lambe & Whitman 1979) and are specifically associated with the deformation of particulate materials. All three processes have been extensively studied in water-saturated sediments under low-stress conditions by soil mechanics engineers and now are being studied using similar experimental methods under the higher-stress conditions that are of special interest in structural geology. The quantification of the deformation mechanisms operating during burial and tectonism of unlithified sediments is important to the understanding of the formation of structures in sediments at shallow depths (sliding, load casts, etc.) and at greater depths through differential compaction and tectonism. This paper presents the basic concepts of this branch of soil mechanics in a theoretical but largely qualitative manner and outlines their role in natural deformation.

It is important to note that the following discussion pertains only to uncemented and weakly cemented sediments. In this context, the term sediment is used to refer to the unlithified state, and not in its widely used form as an abbreviation for sedimentary rock. The authors have also deliberately ignored any discussion of the influences of diagenetic processes on these deformation mechanisms. This subject area is the topic of another paper Addis & Jones (in press) in which we have drawn two general conclusions. Firstly, that dehydration reactions tend to produce overpressures which facilitate the soft sediment deformation mechanisms described here. Secondly, pressure solution, phase changes which do not involve dehydration reactions, and the precipitation of a cementing phase in the pore spaces of a sediment, all contribute substantially to lithification and quickly serve to prevent further deformation by particulate mechanisms.

SEDIMENT COMPRESSION AND CONSOLIDATION

Mechanical dewatering (compaction) of a sediment principally depends upon consolidation, the time dependent generation of a volumetric strain in response to a change in the applied stress (Fig. 1), and compression, the increase in this volumetric strain which accompanies

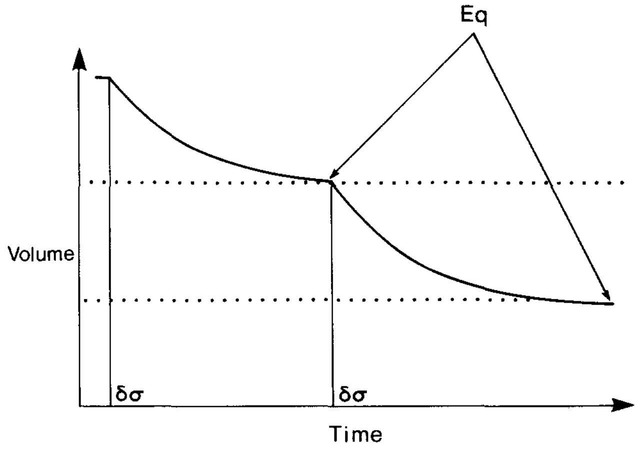

Fig. 1. Idealized volume change record for a sediment consolidating in response to the application of two increments of stress ($\delta\sigma$). The point E_q, represents the equilibrium volume corresponding to the end of primary consolidation.

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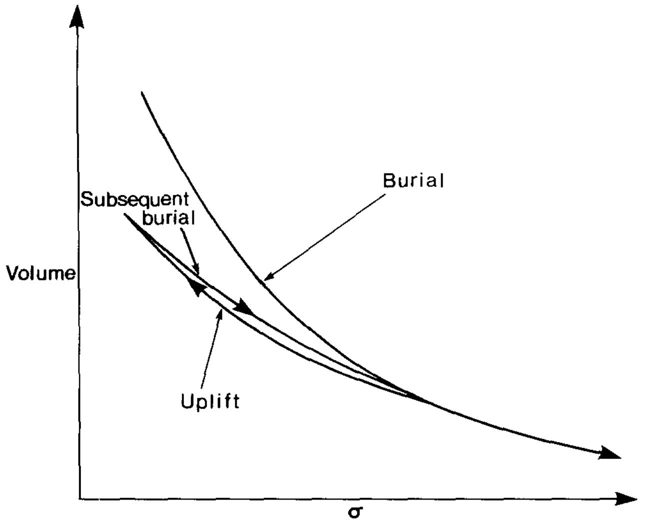

Fig. 2. An idealized burial path for a sediment showing one cycle of burial, uplift and subsequent burial. Increasing depth is represented by increasing total stress (σ) and the volumetric strain by the change in volume.

increasing stress at complete consolidation (Fig. 2). Compaction of a sediment thus results from the expulsion of pore fluid and associated grain-boundary sliding in response to increases in the applied load. This volumetric deformation is therefore associated with a loss of porosity, the development and dissipation of excess pore fluid pressures and a reduction in volume. The process can be described by a complex stress/strain/strain rate relationship based upon theoretical and experimental work (Terzaghi 1943, Taylor 1948, Gibson *et al.* 1967, Been & Sills 1981), and is most simply expressed in the one-dimensional solution of the Terzaghi consolidation equation (Taylor 1948):

$$C_v \frac{\partial^2 u_e}{\partial z^2} = \frac{\partial u_\epsilon}{\partial t} - \frac{\partial \sigma_v}{\partial t}, \tag{1}$$

where C_v = the coefficient of consolidation,
= $[k(1+e)/\gamma_w a_v] = [k/\gamma_w m_v]$;
u_e = the excess pore fluid pressure caused by the change in total vertical stress σ_v;
k = Darcy's coefficient of permeability;
e = the void ratio;
a_v = the coefficient of compressibility;
m_v = the coefficient of volume change
= $m_v = \delta e/\delta\sigma'$ (δe = an increment of volumetric strain; $\delta\sigma'$ = an incremental change in effective stress responsible for δe);
z = the drainage path length;
t = the elapsed time since the application of $\partial\sigma_v$; and
γ_w = the unit weight of water.

From eq. (1), it can be seen that the extent to which a sediment is consolidated by the applied stress will determine the magnitude of any excess pore-fluid pressure developed in the sediment and, thus, the magnitude of

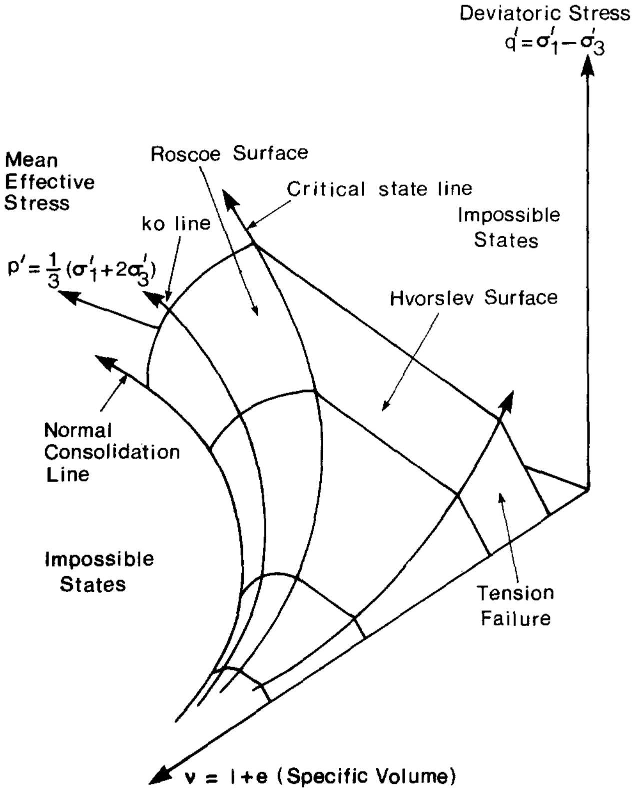

Fig. 3. The general form of the critical state surface in volumetric strain/stress invariant space. The normal consolidation line corresponds to increasing isotropic effective stress ($\sigma'_1 = \sigma'_2 = \sigma'_3$), the k_0 line to the stress system maintaining a condition of no lateral strain, and the critical state line to failure. All stress paths for normally consolidated sediments are found on the Roscoe Surface whilst stress paths for heavily overconsolidated sediments emerge on the Hvorslev Surface and then are constrained on this surface. The axes of the diagram are the mean effective stress, $p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$; the deviatoric effective stress, $q' = \sigma'_1 - \sigma'_3$ and the specific volume $v = 1 + e$ where e is the void ratio.

the effective stress within that sediment (eq. 2) together with the magnitude of the volumetric strain.

$$\sigma' = \sigma - u, \tag{2}$$

where σ' = the effective stress; σ = the total stress; and u = the pore-fluid pressure.

The amount of compression is a measure of the total volumetric strain caused by consolidation under increasing stress, and, thus, is also a measure of the magnitude of the effective stress (Terzaghi 1936). Both consolidation and compression, together with the associated volumetric strain, should be considered as pre-failure responses of the sediment to the applied stress system. Failure of the material only occurs when critical volumetric strain/stress conditions are exceeded, at which point deformation will proceed, maintaining a constant volume at a constant stress (Atkinson & Bransby 1978).

The pronounced change in deformational behaviour when the failure conditions for the sediment are reached, gives rise to the concept of the critical state, traditionally represented by a curve in stress invariant/volumetric strain space (Fig. 3) (Schofield & Wroth 1968, Atkinson & Bransby 1978). Both sediment burial and tectonic deformation can be represented by lines termed stress-paths within the critical state diagram (Fig. 3) (Jones & Addis 1984, in press). The critical state line representing

Table 1. Values of the critical state constants for various sediments determined under the low-stress conditions used in routine soil mechanics testing (Schofield & Wroth 1968, Atkinson & Bransby 1978)

	London clay	Weald clay	Kaolin	Quartz sand
λ	0.161	0.093	0.26	0.03
T	2.759	2.060	3.767	1.930
M	0.888	0.950	1.020	1.420

The values of T are based on p' being measured in KPa.

failure in Fig. 3 can be projected on to both the effective mean stress/effective deviatoric stress and the effective mean stress/volumetric strain planes. The coordinates of these projections in terms of the effective stress invariants p' and q' and the specific volume v are:

$$q' = Mp' \qquad (3)$$

$$v = T - \lambda lnp' \qquad (4)$$

where

$$q' = \sigma_1' - \sigma_3' \quad (\text{for } \sigma_2' = \sigma_3'); \qquad (5)$$

$$p' = \tfrac{1}{3}(\sigma_1' + 2\sigma_3') \quad (\text{for } \sigma_2 = \sigma_3); \qquad (6)$$

$v = 1 + e$ (where e is the void ratio) which is termed the specific volume.

Values for the constants M, T and λ are given in Table 1 (from Schofield & Wroth 1968). Considering these relationships and the law of effective stress (Terzaghi 1936) it can be seen that the volumetric strain, and therefore both the volume and the porosity of the sediment, are dependent on the magnitude of the effective stresses acting on that sediment. The above discussion would suggest that sediment compaction should always be expressed in terms of effective stresses except when overpressures are being considered, when the differences between the total and effective stress paths for the material should be considered (Atkinson & Bransby 1978).

The total stresses acting within a sediment are the sum of two components, the passive stress field created by the weight of the overlying sediment (Jones & Addis 1984) and the externally applied tectonic stress field responsible for the major deformations associated with the evolving basin containing the sediment. The effective stresses acting at any point within the sediment will be the difference between this total stress field and the pore fluid pressure at that point, and thus both the passive and the tectonic stress systems will affect the porosity and volume of the sediment. The magnitude of the volumetric strain is far more dependent on the mean effective stress (p') than it is on the deviatoric effective stress (q') (Jones & Addis in press). However, the magnitude of the effective stress varies inversely with the pore fluid pressure, and although the total passive and tectonic stress fields increase with increasing depth of burial, this is not necessarily the case for the effective stress. In overpressured sediments, therefore, compaction will not proceed until the overpressure can be dissipated and such sediments will sustain near surface compaction characteristics during burial to depths where tectonic deformation can occur. Theoretically at least, deeply buried overpressured sediments may only compact after tectonic deformation has provided a permeable link to the surface. Under such conditions the sediment will be very weak and will easily accommodate the changing geometries of adjacent normally consolidated or cemented beds.

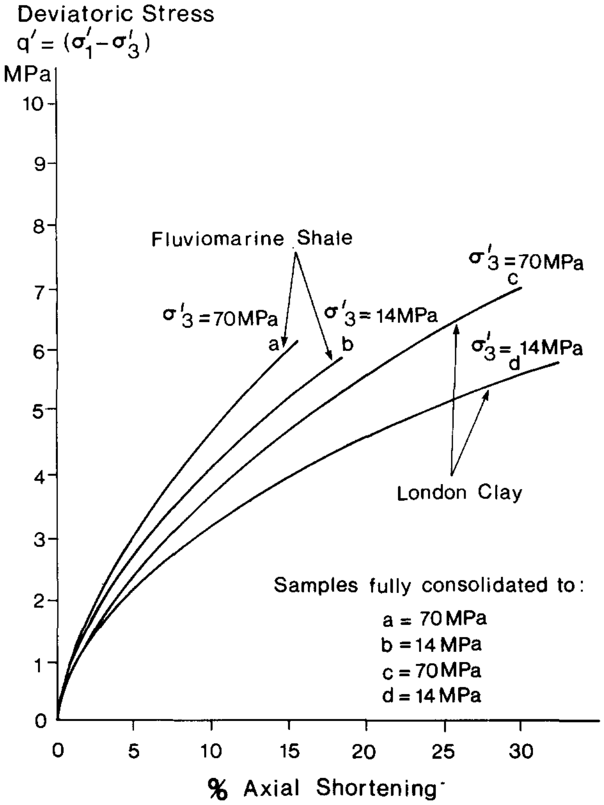

Fig. 4. Stress/strain curves for two clays consolidated and deformed under confining pressures of 14 and 70 MPa.

Even in the absence of overpressures, argillaceous sediments normally consolidated to mean effective stresses equivalent to substantial depths of burial (70 MPa) are found to be easily deformable when axially loaded in triaxial compression with the confining pressure maintained equal to the pre-triaxial deformation consolidation pressure. (See Fig. 4 for data on the London Clay and a fluvio-marine shale.) Specimens recovered from triaxial tests were hard mudrocks in a highly overconsolidated state (Atkinson & Bransby 1978). The deformation recorded as an axial shortening in these experiments was not strictly a failure of the material since the critical state conditions were not exceeded. Thus pre-failure loading of sediments along any stress path corresponding to increasing p' will result in a shape change and a volumetric strain. One particular stress path, the k_0 stress path (Brooker & Ireland 1965, Atkinson & Bransby 1978, Jones and Addis 1984, Jones *et al.* 1984, in press) corresponds to the condition of no lateral strain and thus approximates to the burial stress path for sediments in passive basins. The k_0 stress condition may be expressed as follows:

$$\sigma_H' = k_0\sigma_v', \qquad (7)$$

where σ_H' and σ_v' are the horizontal and vertical effective stresses, respectively and k_0 is the coefficient of earth pressure at rest.

Compression and consolidation *sensu stricto* do not themselves produce macroscopic structures such as folds or shear zones in sediments, but affect the volumetric deformation. They do, however, create shape fabrics in sediments composed of platy minerals, even at small strains. These fabrics strongly influence subsequent deformation and may appear as a cleavage when the sediment has undergone a substantial volumetric strain (e.g. the compaction cleavage in shales). Differential compaction due to variations in lithology or basin geometry may produce structures which will generally have either the geometry of a growth fault or a monocline.

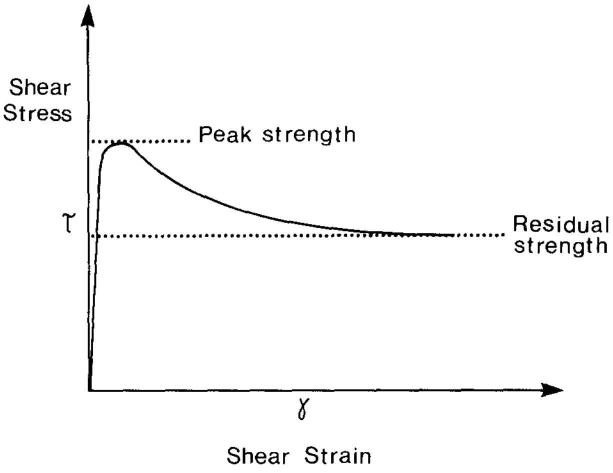

Fig. 5. An idealized shear stress/shear strain curve for an uncemented sediment.

SHEAR DEFORMATION OF SEDIMENTS

In particulate materials, failure is defined as the condition when deformation produces large strains at constant volume and constant stress. This condition occurs when the effective stresses within the sediment attain a value equivalent to any point on the critical state line for that sediment. Subsequent deformation is dominantly through shear, therefore, the relationships between shear stress and shear strain become important, rather than the normal stress/volumetric strain relationships associated with compression.

In the critical state diagram, the effective stress invariant (q') is a measure of the resolved shear stresses acting in a sediment and like the shear stress, is independent of the pore fluid pressure. Excess pore fluid pressures therefore only indirectly influence the shear deformation of a sediment, by reducing the magnitude of the mean effective stress at constant deviatoric effective stress. This hastens the intersection of a stress path with the critical state line. Similarly, if shear deformation permits the dissipation of excess pore fluid pressures in a sediment, p' will increase and shear deformation may cease. The critical state line, therefore, marks the onset of shear failure on an infinite number of drained and undrained stress paths. For normally consolidated sediments, these are found on the Roscoe Surface (Fig. 3) but may follow any path within the critical state space (Fig. 3) or lie on the Hvorslev Surface (Fig. 3) in the case of overconsolidated sediments, that is, those sediments with a more complex burial or tectonic history.

A typical shear stress/shear strain curve for a sediment deformed in a shear box under simple shear conditions exceeding its critical state is shown in Fig. 5. The curve shows a peak shear strength corresponding to the onset of significant shear deformation which then decays to a residual value as the shear strain increases and a shear zone becomes established (Atkinson & Bransby 1978, Lambe & Whitman 1979). The peak shear strength corresponds to a point on the critical state line, and the residual shear strength to the extension of deformation beyond the critical state. It is along the residual shear strength deformation path that most sediment deformation structures will develop. (The term stress path cannot be used for this post-failure deformation because on theoretical grounds, substantiated by experiment, a stress path cannot cross the critical state line). The magnitudes of the peak and residual shear strengths for sediments are related to their cohesion and angles of internal friction (Lambe & Whitman 1979) and are generally greater for saturated sands than for saturated clays. The development of structures by shear deformation may be influenced by lithology, with clays failing before sands.

SOFT-SEDIMENT DEFORMATION

Sedimentary structures resulting from shear deformation (slump folds, gravitational slides, growth faults, etc.) will only be generated when the sediment follows a stress path that intersects its critical state line. Because the critical state concept holds true in environments with mean effective stresses of the order of 100 MPa (and greater?), volume changes, shape changes and textures resulting from sediment compaction, and the structures resulting from shear deformation, will develop in a wide range of environments.

The above discussion suggests that structures produced through sediment compaction and/or deformation cannot be positively assigned to superficial or near surface sedimentary deformation environments or to a deeper tectonic environment, because equivalent effective stress conditions exist over a wide range of depths.

Neither the critical state diagram nor the shear stress/shear strain curve are capable of distinguishing between the components of the passive and tectonic stress field acting on the sediment (or even between the horizontal and vertical principal effective stresses). Thus for uncemented and weakly cemented sediments the term soft-sediment deformation should be used, regardless of environment, to describe those products of deformation brought about by the particulate deformation mechanisms described above; however this can hardly be taken

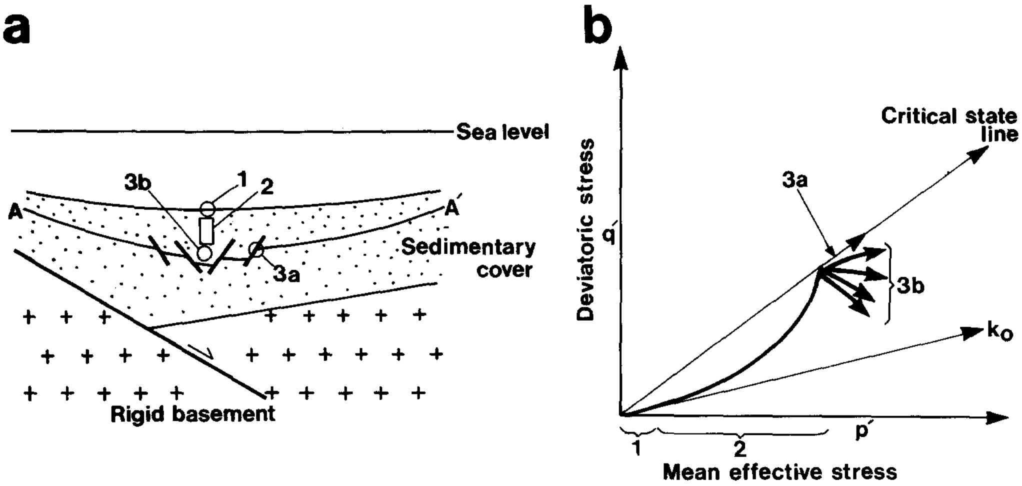

Fig. 6. (a) Cross-section of an idealized extensional basin in which sediments are being subjected to both compactional and extensional deformation. The numbered locations on this section identify representative areas of the sediment body deforming along the different stress paths shown in Fig. 6(b). A–A′ is a representative bedding plane which has become sufficiently deeply buried to be affected by growth faults. (b) Stress path diagram illustrating the stress paths followed by the areas of sediment at the numbered locations shown in Fig. 6(a). 1, Initial burial under k_0 conditions; 2, the increasing influence of the extensional tectonic stress field as the depth of burial increases; 3a, failure of the sediment through the development of a growth fault and subsequent deformation under residual strength conditions; 3b, possible stress paths for the unfailed sediments between the growth faults. If growth fault spacing is to remain constant, this path must return to the k_0 line.

as a complete or satisfactory definition (Maltman 1984).

Under suitable conditions, post critical state phenomena may be restricted to beds for which the constant *M* (Table 1) is small whilst adjacent beds for which *M* is larger will be deforming by pre-failure compaction processes. *M* is generally smaller for clays than for sands which corroborates the observation that clays are generally more severely deformed than adjacent sands in natural sections. Obviously, in thinly bedded multilayers shear deformation of the clays may invoke a passive response from the sands which, in some cases, will deform by other mechanisms (e.g. fracture) to accommodate the shape changes.

Deformation may continue by consolidation/compression and by shear deformation within the same sedimentary unit. If a sediment is subjected to extension, for example, as a consequence of tectonic forces, growth faults may develop that will be initiated as the burial stress path of the sediment intersects its critical state line, and will propagate upwards through the sediment as it accumulates, maintaining an equilibrium condition above which growth fault propagation will cease. On each growth fault, deformation will proceed under residual strength conditions, whilst in the sediment between the faults the passive stress field will still be active and compaction will proceed. This process is represented by the stress path shown in Fig. 6 which shows initial burial along a k_0 stress path becoming modified by the tectonic stress field until failure occurs after which two stress paths remain operative: (1) the residual strength failure deformation path on the growth faults and (2) the pre-failure stress path for the interfault sediments. Because compaction continues during fault motion the curvature of the faults is enhanced (Jones & Addis 1984) and, unless the compaction stress path remains close to the k_0 condition (of no lateral strain), the spacing between the faults must increase as basin extension continues. The tectonic strain rate for the system will be determined by the residual shear strength characteristics of the growth faults and by the difference between the applied stresses and those needed to maintain the k_0 condition. The number and spacing of the growth faults will be a consequence of the dynamic balance between the applied stress systems, the strain rate and the residual strength characteristics of the sediment. Naturally, if the tectonic stress field is reduced in magnitude so that the stress path on the growth faults re-enters the critical state space, displacement on the faults will cease and deformation will continue through compaction.

CONCLUSIONS

Many aspects of sediment deformation can be described and modelled using the principles of soil mechanics outlined in this paper. This approach implies, however, that it is impossible to define soft-sediment deformation in terms of either the time after deposition or depth of burial. More suitable definitions may be possible in terms of the effective stress and/or deformation mechanism.

The generation of soft-sediment structures, and the mechanical compaction of sediments are intimately related processes, driven by both the internal passive stress system and the applied external stresses, such as those that exist during tectonism. The response of a sediment to the applied stresses is primarily dependent on its permeability, and volumetric strain is entirely a consequence of the dissipation of excess pore fluid pressures, that is, it is entirely dependent on the effective stress. Failure of the sediment only occurs once the critical state is exceeded and, thus, depends on the mean and deviatoric effective stresses and on the lithology. The variation of response with lithology often means that soft-sediment deformation may be constrained in certain beds within a succession. These deformation mechanisms are active in any environment where sediments remain unlithified, or only partially lithified, and thus remain weak under their burial stress conditions. In clays, for example, complete consolidation only occurs after fully drained burial to depths exceeding 5 km, and this in the absence of any diagenetic dewatering reactions (Burst 1969, Dunoyer de Segonzac 1970, Rieke & Chilingarian 1974, Addis & Jones in press, Jones & Addis in press).

It is not the intention of this paper to suggest that soft-sediment deformation should be linked entirely with deformation by particulate mechanisms: that would be inappropriate as Maltman (1984) has already shown. However, the extent to which these processes are active in sediments during burial and tectonism is far greater than generally appreciated.

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